Modified Bolotin's Method Applied to Buckling and Vibration of Stressed Plates

S. M. Dickinson*
University of Western Ontario, London, Ont., Can.

In a recent Note by the author,[1] the conventional edge effect method proposed by Bolotin[2] was applied to the buckling and lateral vibration of in-plane loaded, specially orthotropic, rectangular plates. The method involves the assumption that the plate deflection can be described by the product of two sine waves, the plate boundary conditions being satisfied by the introduction of exponential terms having maximum effect at the edges and decaying rapidly away from each edge. It is possible, however, for some combinations of loading conditions, for these additional terms to become oscillatory, thus the basic assumption of the exponential decay of the edge effect terms is violated and the approach fails to yield a solution. A solution can be obtained, however, if a modified version of the Bolotin method (proposed by Vijayakumar[3] and Elishakoff[4]) is used, the application of which is illustrated in this Note.

Modified Bolotin Method

As before,[1] the plate is assumed to lie in the xy-plane, to be bounded by edges $x=0$, a and $y=0$, b, to be of uniform thickness, rectangularly orthotropic material having its axes of symmetry orthogonal to the plate boundaries, and to be acted upon by constant in-plane forces per unit width N_x and N_y (tensile positive) acting in the x and y directions, respectively.

For free vibration, the lateral displacement $w(x,y)e^{i\omega t}$ is governed by the equation

$$D_x\partial^4 w/\partial x^4 + 2H\partial^4 w/\partial x^2\partial y^2 + D_y\partial^4 w/\partial y^4$$
$$-N_x\partial^2 w/\partial x^2 - N_y\partial^2 w/\partial y^2 - \rho\omega^2 w = 0 \qquad (1)$$

where D_x, H, D_y are plate flexural rigidities and ρ is the plate mass/unit area.

It is initially assumed that the plate deflection may be written

$$w=f(x)g(y) = W_0 \sin k_x(x/a-\alpha)\sin k_y(y/b-\beta) \qquad (2)$$

which satisfied Eq. (1), provided that

$$\omega^2 = (1/\rho)[D_x(k_x/a)^4 + 2H(k_xk_y/ab)^2$$
$$+D_y(k_y/b)^4 + N_x(k_x/a)^2 + N_y(k_y/b)^2] \qquad (3)$$

It is then assumed that the plate deflection can be written $w=f(x)\sin k_y(y/b-\beta)$ and that ω^2 continues to be given by Eq. (3). Substitution into plate Eq. (1) yields

$$f(x) = \sum_{j=1}^{4} X_j e^{\gamma_x^{(j)}x/a}$$

where X_j are arbitrary constants and $\gamma_x^{(1,2)} = \pm ik_x$ and

$$\gamma_x^{(3,4)} = \pm[k_x^2 + 2(H/D_x)(k_ya/b)^2 + N_xa^2/D_x]^{1/2} \qquad (4)$$

Received May 16, 1975.
Index categories: Structural Dynamic Analysis; Structural Stability Analysis.

*Associate Professor, Faculty of Engineering Science; (currently on sabbatical leave at the Institute of Sound and Vibration Research, University of Southampton, England).

or, in the more convenient form,

$$f(x) = A_1\cos k_xx/a + A_2\sin k_xx/a$$
$$+A_3\cosh p_xk_xx/a + A_4\sinh p_xk_xx/a \qquad (5)$$

where $p_x = \gamma_x^{(3)}/k_x$. In a similar manner, a function

$$g(y) = \sum_{j=1}^{4} Y_j e^{\gamma_y^{(j)}y/b}$$

is obtained.

Satisfaction of the eight boundary conditions of the plate allows the elimination of the eight coefficients involved in $f(x)$ and $g(y)$, yielding two simultaneous transcendental equations in k_x and k_y, the solution of which enables ω^2 or the buckling loads to be determined from Eq. (3). These equations take a slightly different form from those obtained using the conventional appraoch and usually contain both circular and hyperbolic functions. In the event that $\gamma_x^{(3,4)}$ or $\gamma_y^{(3,4)}$ become imaginary, then the appropriate hyperbolic functions simply convert to circular functions. (In the conventional Bolotin approach, terms of the form

$$Ae^{\gamma_x^{(4)}x/a} \text{ and } Be^{\gamma_y^{(4)}y/b}$$

respectively, are included in the $f(x)$ and $g(y)$ defined in Eq. (2) and it is required that these exponents remain real.)

An example of a problem for which $\gamma_x^{(3,4)}$ becomes imaginary is that of the buckling of an isotropic plate under in-plane force N_x only. Substitution of $\omega^2=0$ into Eqs. (3) and (4) yields

$$\gamma_x^{(3,4)} = \pm i\, k_y^2a^2/k_xb^2$$

Numerical Results

The modified Bolotin approach was applied to the lateral vibration of a clamped plate subject to a) hydrostatic in-plane force ($N_x=N_y$), for comparison with Ref. 1; b) uniaxial in-plane force (N_x varied, $N_y=0$), to illustrate its applicability to a situation where the γ_x values are all imaginary (p_x then becomes imaginary).

The simultaneous equations which result for a clamped plate may be written

$$2p_x + (p_x^2-1)\sin k_x\sinh p_xk_x - 2p_x\cos k_x\cosh p_xk_x = 0$$

$$2p_y + (p_x^2-1)\sin k_y\sinh p_yk_y - 2p_y\cos k_y\cosh p_yk_y = 0$$

$$2p_x' - [(p_x')^2+1]\sin k_x\sin p_x'k_x - 2p_x'\cos k_x\cos p_x'k_x = 0$$

where p_x' is the amplitude of p_x. (When provision is made for the possibility that p is imaginary, care should be taken in the computation of the roots of the simultaneous equations to verify that the quantity p' is not unity for the computed values of k_x and k_y, as this represents the case of repeated roots ($k=p'k$) and Eq. (5) is not then the correct form for the deflection function).

a. Clamped Plate Subject to Hydrostatic In-plane Load

In Table 1 the fundamental frequency parameters for three square orthotropic plates are presented as computed using the conventional and modified Bolotin approaches and a series solution.[5] The table shows that there is very little difference between the results obtained using the two Bolotin approaches, the modified approach yielding very slightly more accurate values.

b. Clamped Plate Subject to Uniaxial In-plane Force

Figure 1 shows a comparison between the square of the fundamental frequency parameter for a square clamped isotropic

Table 1 Fundamental frequency parameter $\omega ab(\rho/H)^{1/2}$ for a square orthotropic clamped plate subject to hydrostatic in-plane force (tension positive)

$D_x/H = D_y/H$		½		1		2	
$Na^2/\pi^2 H$		Bolotin	Series (Ref. 5)	Bolotin	Series (Ref. 5)	Bolotin	Series (Ref. 5)
−2	Conventional (Ref. 1)	15.784	17.742	26.726	28.573	40.900	42.641
	Modified	15.808		26.793		41.080	
0	Conventional (Ref. 1)	27.473	28.071	35.092	35.985	46.832	47.959
	Modified	27.476		35.112		46.915	
10	Conventional (Ref. 1)	54.927	54.981	59.802	59.925	67.913	69.165
	Modified	54.927		59.802		67.917	

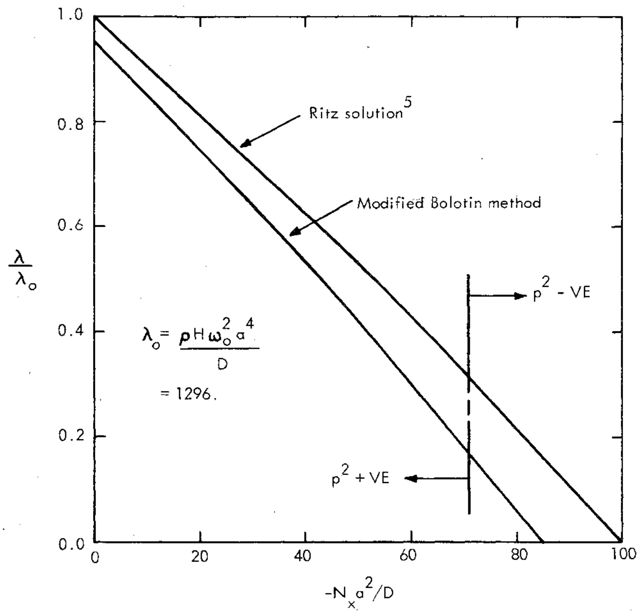

Fig. 1 Frequency parameter squared vs uniaxial in-plane force for square isotropic clamped plate.

plate under in-plane force N_x only, as computed using the modified Bolotin approach, and a Ritz approach.[6] Only the region in which N_x is compressive is shown to illustrate the performance of the modified Bolotin approach when $\gamma_x^{(3,4)}$ is imaginary, the region in which this occurs being indicated. As would be expected, there is no significant change in the nature of the curve as this region is entered.

It should be recognized that the performance of either Bolotin approach is relatively poor in the compressive in-plane force domain and that much more accurate results are obtained for tensile loadings.

References

[1] Dickinson, S.M., "Bolotin's Method Applied to the Buckling and Lateral Vibrations of Stressed Plates," *AIAA Journal*, Vol. 13, Jan. 1975, pp. 109-10.

[2] Bolotin, V.V., "An Asymptotic Method for the Study of the Problem of Eigenvalues for Rectangular Regions," *Problems of Continuum Mechanics (Volume Dedicated to N.I. Muskhelishvili), Society of Industrial and Applied Mathematics*, Philadelphia, Pa. 1961, pp. 56-68.

[3] Vijayakumar, K., "A New Method for Analysis of Flexural Vibration of Rectangular Orthotropic Plates," *Journal of the Aeronautical Society of India*, Vol. 23, 1971, pp. 197-204.

[4] Elishakoff, I.B., "Vibration Analysis of Clamped Square Orthotropic Plate," *AIAA Journal*, Vol. 12, July 1974, pp. 921-924.

[5] Dickinson, S.M., "The Flexural Vibration of Rectangular Orthotropic Plates Subject to In-plane Forces," *ASME Transactions: Journal of Applied Mechanics*, Vol. 38, Sept. 1971, pp. 699-700.

[6] Bassily, S.F. and Dickinson, S.M., "Buckling and Lateral Vibrations of Rectangular Plates Subject to In-plane Loads—A Ritz Approach," *Journal of Sound and Vibration*, Vol. 24, Sept. 1972, pp. 219-239.

Three-Dimensional Laminar Boundary Layer in Low-Speed Swirling Flow with Mass Transfer

Margaret Muthanna* and G. Nath†
Indian Institute of Science, Bangalore, India

SWIRLING flow occurs in rockets, jet engines, vortex valves, industrial furnaces, and many other types of machinery. In view of such a wide application of swirling flow, it is necessary to have a thorough understanding of such flows, if their design is to be accomplished on any kind of rational basis. Recently, Lewellen[1] and Murthy[2] made an extensive survey of swirling flows and their applications. The strong interaction that exists between the boundary layer and the outer flow in the case of rotating flows has been discussed by Rott and Lewellen.[3] The similarity solutions are quick and reliable solutions of these problems, which, if solved exactly, would require considerable manpower and computer time.

Back[4] has obtained the similarity solutions for low-speed three-dimensional laminar compressible boundary layer with swirl and without mass transfer on an axisymmetric surface of variable cross section. But, in his analysis, he employed the simplifying assumption that the density-viscosity product $\rho\mu$ is constant in the boundary layer and the Prandtl number Pr is unity.

Our objective in this study is to obtain similarity solutions of the above problem for a perfect gas, employing realistic gas properties ($\rho \propto H^{-1}$, $\mu \propto H^{\omega}$, $Pr = 0.7$; where H and ω are the enthalpy and exponent of viscosity, respectively) together with mass transfer. We have clearly displayed the inadequacy of solutions obtained under the simplifying assumptions of $\omega = Pr = 1$. We have used successfully the method of parametric differentiation in combination with quasilinearization to solve the governing equations.

Received May 7, 1975; revision received July 15, 1975.

Index category: Boundary Layers and Convective Heat Transfer-Laminar.

*Research Student, Department of Applied Mathematics.

†Associate Professor, Department of Applied Mathematics.